

Fundamentals of statistics

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Because of economic reasons, the statistical capture of the measurement stability is very important during the debug and acceptance phase of a test program. Some of the methods are included in the standard scope of Tecap. The following application note AN. Tecap01.2012.en describes the definitions and how they are available.

Tecap offers the numerical statistics function per mouse click!

cp?..cpk?..cgm?...cgm?...

Today statistical methods are necessary for the capture of the test quality, because undesirable variations in the quality process are always occurring. Everybody knows the bell curve of the Gaussian distribution. It describes the random distribution of random measurement values between +infinite and -infinite. The knowledge of the distribution is not sufficient for the analysis of the manufacturing process. The results must be set in reference to the limit values.

How are these values established?

Tecap offers functions for the process optimization, with which the user is able to access these information and to deduce the correct actions from them.

The described indices are referring to meaningful measurement values limits and specifications and therefore have to be determined not by the test engineer.

Repeatability

Often the repeatability is more important than the absolute correct measurement value. The repeatability shows that the measurement is stable. It is easy to determine which influencing factors can distort the measurement result.

Tecap is embedding the statistical calculation base in the test procedure. Man has the absolute and in any moment traceable system control.

Process capability

Unstable measurements, wrong measurement ranges of the instruments, unrealistic measurement limits can not be used as a base for test result analysis.

Measurement equipment capability

Each system, each test program should improve and increase the profitability in the production. Unstable measurements result in increased failure, because limits have to be set wider and that way the yield is respectively reduced. The more accurate and precise the measurement is, the more tight the limits can be set and the yield increased.

Therefore the use of statistical methods did not only originate from the fantasy of quality gurus, but have a serious economic background.

The user of Tecap does not need to have advanced fundamental knowledge – all important reportings are available per mouse click!

Statistical reportings like Average, Standard Deviation and Variance are embeded in Tecap

- Process capability index cp
- Product process capability index cpk
- Measurement equipment index cgm
- Measurement equipment index Golden Device cgmK

The statistical values are displayed directly online during testing in the menu **Testfloor**. Quality deviations of the Device Under Test or erroneous function of the test system are recognized immediately.

The screenshot shows the 'Testfloor' application window with a 'Numerical Statistics' tab. It displays two tables of statistical data for five test groups. The top table shows individual test results, and the bottom table shows overall statistics.

Test Group	Test ID	Name	Min. Value	Max. Value	Mean Value	Std.Dev.	cp	cpk	cgm	cgmK	
1	Power (FCT.1)	1.1	Voltage	14.3 V	15.9 V	14.993	0.4501	3.70	3.70	0.74	0.74
2	Power (FCT.1)	2.1	Current	26.3 mA	33.6 mA	30.040	2.0867	3.19	3.19	0.64	0.63
3	Power (FCT.1)	3.1	R1	4.67 kOhm	4.73 kOhm	4.693	0.0142	3.31	3.15	0.66	0.66
4	Power (FCT.1)	4.1	R2	247.8 Ohm	252.4 Ohm	250.427	1.4563	2.86	2.76	0.57	0.56
5	Power (FCT.1)	5.1	VoltageOut	24.8 V	25.2 V	24.996	0.1430	2.91	2.91	0.58	0.55

Test Group	Test ID	Name	Min. Value	Max. Value	Mean Value	Std.Dev.	cp	cpk	cgm	cgmK	
1	Power (FCT.1)	1.1	Voltage	14.3 V	15.9 V	14.871	0.4476	3.72	3.63	0.74	0.73
2	Power (FCT.1)	2.1	Current	27.9 mA	33.6 mA	30.313	1.8871	3.53	3.48	0.71	0.71
3	Power (FCT.1)	3.1	R1	4.68 kOhm	4.73 kOhm	4.697	0.0138	3.41	3.33	0.68	0.67
4	Power (FCT.1)	4.1	R2	248.1 Ohm	251.9 Ohm	250.422	1.1476	3.63	3.51	0.73	0.73
5	Power (FCT.1)	5.1	VoltageOut	24.8 V	25.2 V	24.918	0.1171	3.56	3.33	0.71	0.70

Average, Standard Deviation and Variance

v..... Test value
 exec_cnt Test counter
 sums Sum of test values
 sqrs Sum of the squared test values
 average..... Average
 variance Variance
 std_dev Standard deviation (only for exec_cnt>1, because division by 0 is not defined)

$$\text{sums} = \sum v$$

$$\text{sqr} = \sum v^2$$

$$\text{average} = \text{sums} / \text{exec_cnt}$$

$$\text{variance} = \frac{\text{sqr} - (\text{sums} / \text{exec_cnt}) * \text{sums}}{\text{exec_cnt} - 1}$$

$$\text{std_dev} = \sqrt{\text{variance}}$$

Process capability index cp

The Cp value shows the ratio between the specified tolerance and the process spread. A value of >2 is to aim for.

The Cpk value incorporates the position of the mean value compared to the tolerance center. Therefore the Cpk value is always smaller than the Cp value.

hi_limit.....	high limit
lo_limit.....	low limit
tolerance.....	test tolerance
std_dev.....	standard deviation

$$\text{tolerance} = \text{hi_limit} - \text{lo_limit}$$

$$\text{cp} = \frac{\text{tolerance}}{6 * \text{std_dev}}$$

Process capability index cpk

The Cpk value is documenting the production process capability.

It shows which tests will have a good yield in the production. Each test has an assigned Cpk value, it reflects the quality of the lot. The mean value must be within the tolerance defined by the high and low limit. The cpk value is called „critical“ or „asymmetrical“ process capability, because the mean value is taken into consideration. The quality of the process is good in case of $\text{cpk} > 1.33$.

hi_limit.....	high limit
lo_limit.....	low limit
tolerance.....	test tolerance
std_dev.....	standard deviation
average.....	average
variance	variance
cpk	cpk (capability process key)

$$\text{cpk_low} = \frac{\text{average} - \text{lo_limit}}{3 * \text{std_dev}}$$

$$\text{cpk_high} = \frac{\text{hi_limit} - \text{average}}{3 * \text{std_dev}}$$

if (cpk_low < cpk_high)

 cpk = cpk_low

else

 cpk = cpk_high

Measurement equipment capability index cgm

The cgm value (capability gauge measurement) represents the influence of the measurement arrangement on the measurement value distribution.

The distribution is good in case of $\text{cgm} > 1.33$.

- Execution of the measurement
- The measurement equipment must be calibrated
- The measurement equipment can not be recalibrated during the measurement
- 1 Golden Device as a reference
- Test series with 25 ... 50 measurements

hi_limit.....	high limit
lo_limit.....	low limit
tolerance.....	test tolerance
std_dev.....	standard deviation
variance	variance
cgm	cgm (capability gauge measurement)

$$\text{tolerance} = \text{hi_limit} - \text{lo_limit}$$

$$\text{cgm} = \frac{0.2 * \text{tolerance}}{6 * \text{std_dev}}$$

Measurement equipment capability index cgm_k

The value cgm_k represents the influence of the measurement equipment on the distribution. The index cgm_k incorporates the nominal value of the test.

Cgmk is important for the correlation tests between systems in relation to a Golden Device.

hi_limit.....	high limit
lo_limit.....	low limit
tolerance.....	test tolerance
std_dev.....	standard deviation
variance	Varianz variance
cgmk.....	capability gauge measurement
average.....	average value
golden_dev	target value of the Golden Device

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tolerance = hi_limit - lo_limit

golden_dev = ( hi_limit + lo_limit ) * 0.5

cgmk1 = 
$$\frac{( \text{golden\_dev} + 0.1 * \text{tolerance} ) - \text{average}}{3 * \text{std\_dev}}$$


cgmk2 = 
$$\frac{\text{average} - ( \text{golden\_dev} - 0.1 * \text{tolerance} )}{3 * \text{std\_dev}}$$


if (cgmk2 < cgmk1)
    cgmk = cgmk2
else
    cgmk = cgmk1
```

Summary

All statistical values are necessary in the assessment of the test process quality and are necessary to release the test program, test system, product as well as the process control. Tecap provides online all this information.

In addition to the statistical functions selectable per mouse click, Tecap offers additional outstanding functions:

- Trend analysis
- Integrated data base
- Routing and Connections
- Intuitive operator interface
- Multi DUT parallel test
- Test plan development
- Easy to use instrument system
- Overall concept – out of the box -